Implementation and Experimental Results of Superposition Coding on Software Radio

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Abstract—In theory, multi-user techniques such as superposition coding (SPC) are known to improve throughput in wireless networks. However, in order to understand their practical limitations, it is imperative to actually implement and experiment with such techniques in a realistic setting. In this paper, we present the design of a software radio-based implementation of SPC using the GNU Radio architecture. We describe software and hardware issues associated with SPC implementation on a Universal Software Radio Peripheral (USRP) board. We then experimentally evaluate the performance of this SPC system and demonstrate how it compares to a time-division multiplexing (TDM) system.

Keywords- Software Radio, GNU Radio, USRP, Superposition Coding, OFDM, Time-Division Multiplexing.

I. INTRODUCTION AND MOTIVATION

The past few decades have witnessed a surge of revolutionary research developments in the area of point-to-point wireless communication. Notable contributions include near-capacity achieving low-density parity-check (LDPC) and turbo codes and other breakthroughs such as multiple-input multiple output (MIMO), orthogonal frequency division multiplexing (OFDM), and code-division multiple access (CDMA) that have found widespread practical utility. In order to meet the ever-growing demand for wireless services, there has also been an increased research focus on multi-user communication techniques that may form the basis for next-generation wireless technologies. While theoretical results in this area hold the promise of significant performance gains [1], they often assume analytically tractable models for wireless channels, and in addition, perfect synchronization and error-free feedback. However, such assumptions are usually not tenable in practice. This motivates the experimental study of multi-user communication schemes that are well-studied only in theory.

As a first step in this direction, we experimentally evaluate the performance of superposition coding (SPC) [1], a broadcast multi-user scheme. We choose to study superposition coding for the following reasons:

1) SPC is known to achieve capacity in the degraded Gaussian broadcast channel.

2) The theory of SPC is well developed. Theoretically, its architecture is easily scalable to support more users, and consequently, higher throughput.

3) SPC forms the gateway to more sophisticated networking scenarios and experiments such as SPC-based MAC layer design.

Conventionally, practical testing of a theoretical idea like the SPC technique would require the design of new hardware which would involve a considerable design effort. Such a long development cycle, however, is not conducive to rapid prototyping and verification of new protocol designs. An alternative design paradigm is the so-called software defined radio (SDR) [2], where the design flow is mostly in software, thus making it useful for fast prototyping of new communication techniques by leveraging the inherent flexibility of software-based systems compared to their hardware-based counterparts.

GNU Radio is an open source software development toolkit that provides the signal processing blocks to implement SDRs and it interfaces with a Universal Software Radio Peripheral (USRP) board that serves as an analog and RF front-end [3]. Using GNU Radio as a cognitive radio platform [4]–[7] or a prototyping system [8], [9] became popular in the past few years. Some notable examples of USRP-based testbeds are UT Austin’s Hydra [10] and the mesh network platform by Bell Labs and Microsoft Research [11]. Recently, the MAC layer for an SPC-based physical layer was implemented [12]. However, to the best of our knowledge, the USRP implementation of the SPC PHY layer is yet to be investigated.

In this paper we describe the realization of OFDM-based SPC on GNU Radio and the study of SPC through experiments. We focus on using GNU Radio/USRP as a platform to evaluate and compare the performance of SPC and a time-division multiplexing (TDM) system.

II. SUPERPOSITION CODING

A. Theoretical View

Consider a base station $B$ that wishes to independently communicate with two users $N$ (a “near” user) and $F$ (a “far” user) over AWGN channels. Assume that the (point-to-point) channel between $B$ and the near user $N$ has a higher capacity than from $B$ and $F$. Such a scenario can arise, for example, in a cellular system where one user is close to the base station and the other near the cell edge. One of the key questions is: What is the communication scheme that can achieve the maximum possible transmission rates to both users?
In information theory, this question is answered by formulating the problem as that of communication over degraded broadcast channels [1]. The capacity region, which is the set of all simultaneously achievable rates for both users, is known. All these rates can be achieved using superposition coding, i.e., the superposition (and simultaneous transmission) of the encoded messages of all the users. The idea is as follows: allocate most of the power to users with bad channels to the base station. Users with better channel capacities can always decode messages meant for those with poorer channels and can thus effectively cancel interference from those messages in their received signal(s). A brief description of the decoding strategy follows; details can be found in [1].

Denote by \( X_N \) (or \( X_F \)) the encoded near (or far) user’s signal, each with unit power. Let the total power of the base station \( B \) be unity, of which a fraction \( \alpha \) (resp. \( 1 - \alpha \)) is given to the far (resp. near) user. Since the channel from \( B \) to \( N \) is AWGN, the near user observes \( Y_N = \alpha X_F + (1 - \alpha) X_N + W_N \) where \( W_N \) denotes WGN with variance \( \sigma^2 \). Since the channel from \( B \) to \( F \) has a higher capacity than that from \( B \) to \( N \), the near user \( N \) can decode the far user’s message. After canceling the interference \( \alpha X_F \), the near user observes \( Y_N - \alpha X_F = (1 - \alpha) X_N + W_N \) which can be used to decode the near user’s message. The far user, on the other hand, can only decode its own message but not the near user’s message. Since the far user has the worse channel, most of the power is allocated to it. Thus in general, \( \alpha > \frac{1}{2} \).

B. BER versus the Power Allocation Parameter \( \alpha \)

In this subsection, we analytically derive the bit-error-rates (BER) of the near and far users with respect to the power allocation parameter \( \alpha \), which allows us to optimally design the SPC-based system in terms of error performance. To this end, we consider the scenario in which both the users use BPSK as their signal constellations. While we observe that the error rate for the far user monotonically decreases with increasing \( \alpha \), we shall see that the geometry of the modulation constellation diagram plays a key role in shaping the BER for the near user. In fact we will show that when \( \alpha > \frac{1}{2} \), setting \( \alpha = \frac{1}{2} \) is always optimal for the near user. See the Appendix for the derivation and discussion. Fig. 1 shows the behavior of bit-error-rate (BER) versus the power allocation parameter for different signal-to-noise ratio (SNR) values for both the near and far users, respectively (see (3) and (4) in the Appendix).

In essence, the BER for the near user is minimized (for \( \alpha > \frac{1}{2} \)) when the constellation points are arranged as on a (finite) lattice grid, which ensures equal spacing between adjacent points. For instance, when both the near and far user users employ QPSK, the BER for the near user is minimized when the overall signal constellation is 16-QAM. This may be visualized as four 4-PAM constellations. Now, since the optimal power allocation parameter is independent of the transmission power, the optimal power allocation problem simply reduces to the case where both near and far users use BPSK. Thus, the optimal power allocation parameter is \( \alpha = \frac{1}{2} \) also for the scenario where both the near and far user signal constellations are QPSK. One might extend this argument to reason out that the optimal power allocation is \( \alpha = \frac{4}{5} \) even in the general case wherein the near and far users’ constellations are \( 2^{2M_1} \)- and \( 2^{2M_2} \)-QAM respectively, for arbitrary \( M_1 \geq 1 \) and \( M_2 \geq 1 \).

III. SPC Transceiver Implementation

In this section, we will briefly describe the GNU Radio/USRP platform and the transceiver design of a practical superposition coded system. Due to the space limit, readers who are interested in the system design and implementation details may refer to [13].

A. Platform

Our SPC transceiver is implemented on the GNU Radio/USRP platform. The main functionality of the USRP is to act as the RF front-end, perform digital up/downconversion of the signal, and communicate with the PC via Ethernet. All the signal processing is software-based and is completely performed in the PC, which makes this platform extremely flexible. For the signal processing of the digital samples on the PC, we use the increasingly popular GNU Radio architecture [3], which is a collection of open source libraries and a scheduler, and provides drivers (API) to connect and operate the USRP. The GNU Radio platform offers significant advantages in terms of rapid prototyping and the availability of built-in DSP libraries.

B. Transceiver Design

Our SPC system uses OFDM [14] to transmit data over multiple orthogonal subcarriers. The low transmission rate per subcarrier, coupled with the orthogonality of the subcarriers, greatly simplifies system design, as evidenced by its widespread use in local- (IEEE 802.11) and wide- (IEEE 802.16) area networks. The main benefits of using OFDM include the relative ease of channel estimation, equalizer...
design, and code design. Moreover, OFDM systems offer a high degree of bandwidth scalability. In our system, SPC is implemented over each subcarrier.

In the transmitter, the payload pair (near and far user data in bits) is provided to the physical layer by the higher layers. The source bits are encoded by a rate 1/2, \( G = [133, 171] \) convolutional code. The signal constellation mapper then multiplexes the coded bits intended for the two users and maps it to the superposition constellation. Some of the tones in OFDM are used as pilots, and a usage map is employed to specify the tones over which the payload may be transmitted. Finally, the OFDM modulator inserts the preamble (used for channel and frequency synchronization), modulates the SPC symbols onto 16 subcarriers, and then inserts the cyclic prefix (CP).

In the receiver, the USRP hardware is connected to the receiver path, which comprises the timing and frequency synchronization block (doing coarse and fine frequency tracking), the OFDM demodulator, the usage demapper, and the SPC demapper. The output bits are then decoded using a Viterbi decoder. Timing and frequency synchronization are achieved with the Schmidl-Cox algorithm [15].

![Packet structure](image)

The packet structure is depicted in Fig. 2. The preamble sequence is used primarily for frequency and timing synchronization and is designed by repeating a pseudo-random training sequence of length 24 symbols (TS1) twice. The channel estimation symbols are used for performing equalization and are generated by repeating a pseudo-random sequence of length 16 symbols (plus CP) three times. The length of the cyclic prefix was chosen to be 4 symbols (4 \( \mu s \)).

![Depiction of the relative locations of the radios in the implementation set-up](image)

Figure 3. Depiction of the relative locations of the radios in the implementation set-up. These distances were chosen so that both the receivers lie in the far-field of the transmitter and observe different SNRs.

IV. EXPERIMENTAL RESULTS

In this section, we provide experimental results that quantify the performance of the SPC system. First, we provide the packet-error-rate (PER) versus SNR performance of the GNU-radio based implementation of the SPC system, and then we compare the performance with the TDM-based system. Since we work on the packet-based system, the PER, not the BER, is the relevant metric.

The experimental setup consists of a transmitter and two (near and far) receivers. Each transceiver is a software radio along with a USRP2 and a PC. Fig. 3 shows the experimental setup with the relative locations of the three USRP2’s. The system parameters are the following. The carrier frequency is set to 903 MHz and the bandwidth is 1 MHz. Both the constellation of the near and far user are set to BPSK. The payload size is 508 bytes plus 32-bit CRC for both the near and the far user, so the coded packet length is 1024 bytes.

A. PER versus SNR

Fig. 4 plots the PER versus SNR performance of the SPC system for both the near and far users when \( \alpha = \frac{4}{5} \). The experiment was conducted indoors in our laboratory. Since more power is allocated to the far user, the far user has better PER. The figure shows that our SPC system gives very good performance (PER achieves \( 10^{-2} \)).

![PER versus SNR](image)

Figure 4. PER versus SNR for the near and far users for \( \alpha = \frac{4}{5} \). Note that the SNR is defined as the measured preamble power divided by the noise power. Since the far user is allocated more power than the near user, its PER is lower. Both the far and the near user achieve a good performance (PER<\( 10^{-2} \)) at moderate SNR.

B. Optimal \( \alpha \)

Recall that Fig. 1 shows theoretically how the BER varies with the power allocation parameter \( \alpha \) and that the optimal value of \( \alpha \) for the near user is \( \frac{4}{5} \). When the BER is optimized, the PER is also optimized, and additionally, the PER is a non-decreasing function of BER. Therefore, Fig. 1 gives an idea of how the PER should behave under different \( \alpha \) and different SNR. Fig. 5 shows how the PER varies with \( \alpha \) experimentally for different transmitter gain factors. The optimal \( \alpha \) for the near user is indeed around 0.8 and for the far user, the PER decreases with increasing \( \alpha \) or increasing transmitter gain factor.

\(^1\)In GNU Radio, the transmission power is adjusted by setting the transmitter gain factor using the function `subdev.set_gain()`. Ideally, the radiated power is proportional to the square of the transmitter gain factor.
C. SPC versus TDM

Our setup allows for easy switching between SPC and TDM. Indeed, to work as a TDM system, we just set either $\alpha = 0$ or $\alpha = 1$. To compare the SPC to the TDM system, we assume the near and the far user acquire an equal amount of data and the locations of the near and far users are fixed. For each experiment, we choose a transmission power $P$ (by setting the transmitter gain factor in the GNU Radio). For the TDM system, we use this transmission power $P$ to send packets to the near user in the first time slot and then to the far user in the second time slot. For the SPC, at each time slot, we simultaneously send packets to the near and the far user. The total transmission power is $P$, so the power allocated to the far and near user is $\alpha P$ and $(1 - \alpha)P$, respectively. We repeat the experiments several times by choosing a larger $P$, but at the same time increasing the value of the power allocation parameter $\alpha$ such that the near user always gets a fixed allocated power.

The goodput per time slot is computed and the results are shown in Fig. 6. It shows that once the transmitter gain factor is above 0.0945 (in this particular setup), by allocating a small portion of the power to the near user and sacrificing little performance of the far user, both the near and the far user goodput (and thus the overall system goodput) can be significantly boosted. One may argue that SPC is similar to using a higher constellation, so if the TDM system adopts a higher constellation, the performance will be better. If doing so, then we can also change the constellation of SPC, and still get a performance gain.

V. LESSONS LEARNED

We now briefly overview some issues involved in implementing the SPC system using GNU Radio.

- The GNU Radio scheduler was designed for a flow-based framework, i.e., each block operates on a stream of data rather than packets of data. This makes the design of a frame-based system quite challenging. Moreover, implementing feedback loops between signal processing blocks is cumbersome. One solution is to implement all the functions in one big GNU Radio block, but this sacrifices the reusability of the functional blocks. From our experience, implementing all the functions in one GNU Radio block significantly simplifies the design.

- The effective bandwidth of the USRP1 (the first generation USRP) is much smaller than that set by the user. The cause of this problem lies in the highly non-ideal transmit path implementation of the USRP1. The DAC’s on the transmit path are designed to operate at a fixed frequency of 128 MHz [3]. Therefore, any digitally synthesized signal at a lower bandwidth to be input to the DAC’s must be interpolated to 128 MHz. However, we observe that the USRP1 uses a rather simplistic scheme to implement this interpolation, so it shows a poor passband response. Such a frequency response causes significant degradation of subcarrier SNR as one moves away from the DC subcarrier. Similar problems were reported recently elsewhere [16]. The solution is to implement a software-based filter and do part of the interpolation in software.

VI. CONCLUSION AND FUTURE WORK

In this paper, we have investigated the design of an OFDM-based SPC transceiver on the GNU Radio/USRP2 platform and we have identified design issues in implementing SPC on this platform. We have analytically derived and experimentally verified the optimal power allocation parameter. Additionally, we have demonstrated a broadcasting scheme using SPC and perform experiments to compare the performance with TDM. We have also shown that when the transmission power is above a certain level (depending on the channel condition and the setup), the SPC system gives higher goodput per time slot than the TDM system. Our future work will include the implementation of medium-access and routing protocols for superposition coding.
\[ P_b = \frac{1}{2} \left( 2Q(\sqrt{\gamma_1}) - Q(\sqrt{\gamma_1} + \sqrt{\gamma_2}) + Q(\sqrt{\gamma_1} + \sqrt{\gamma_2}) - Q(2\sqrt{\gamma_1} - \sqrt{\gamma_2}) \right). \]

Now, for the far user, the decision regions are simply the two sides of the \( y \)-axis (i.e., \( x \geq 0 \) and \( x < 0 \)). Using a similar technique as before, we obtain

\[ P_{b,f} = \frac{1}{2} \left( Q(\sqrt{\gamma_1} + \sqrt{\gamma_2}) + Q(\sqrt{\gamma_1} - \sqrt{\gamma_2}) \right), \]

where \( \gamma_1 = \gamma_f \sqrt{1 - \alpha} \) and \( \gamma_2 = \gamma_f \sqrt{\alpha} \), and \( \gamma_f \) is the SNR at the far user.

We make the following observations.

- Evidently, setting \( \alpha = 0 \) and \( \alpha = 1 \) is optimal (BER-wise) for the near and far users respectively.
- For the far user, \( P_{b,f} \) monotonically reduces with increasing \( \alpha \). This is because the higher \( \alpha \) is, the farther away are the constellation points from the origin. For the near user however, the geometry of the constellation plays a critical role in determining \( P_b \). Indeed, for \( \alpha = 0 \), the points 1 and 3 (and points 2 and 4) exactly overlap. As \( \alpha \) increases from 0, \( P_b \) also increases (since points 2 and 3 approach each other) and attains a local maximum (visually found to occur at \( \alpha \approx 1/\sqrt{10} = 0.3163 \)). As \( \alpha \) increases further, points 2 and 3 move away from each other and \( P_b \) reduces. Note however that as \( \alpha \) approaches 1, the distances between points 1 and 2 (and 3 and 4) also get smaller, and \( P_b \rightarrow \frac{1}{2} \) as \( \alpha \rightarrow 1 \).
- In a practical scenario (when \( \alpha > \frac{1}{2} \)), the BER for the near user is always minimized at \( \alpha = \frac{4}{5} \) theoretically. This can be intuitively explained from the geometry of the constellation. Indeed when \( \alpha \) is large, the points 1 and 2 (and points 3 and 4) are close to each other since \( 1 - \alpha \) is small. Alternatively, when \( \alpha \) is small, the points 2 and 3 are close to each other. Thus, \( P_b \) is minimized when the distances between all the points are the same, i.e., when \( \sqrt{\alpha} - \sqrt{1 - \alpha} = \sqrt{1 - \alpha} \), so we obtain \( \alpha = \frac{4}{5} \) as the local minimum.

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